Flatland: A Romance of Many Dimensions

INSTRUCTOR’S Manual

Josephe Zakhary
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Flatland: A Romance of many Dimensions – A Book Review

Every 1,000 years, a Sphere visits Flatland to find a new apostle to spread the word about a third dimension. This time A. Square is the chosen apostle. A. Square dutifully pens this memoir in an attempt to convince the other inhabitants of Flatland as to the existence of a third dimension.

Flatland’s inhabitants are comprised of various geometric shapes, each representing a class of inhabitants in Flatland’s hierarchy. The lowest class, Isosceles Triangles, consists of soldiers and workmen, such as the infantry and policemen. When the Isosceles Triangles reproduce, their offspring possess half a degree more than their parents. Every Isosceles Triangle hopes that their offspring will move up a step within the hierarchy of Flatland to become a member of the middle class.

The middle class is made up of Equilateral Triangles. These triangles are the traders.

The professional class is comprised of Squares and Pentagons. These shapes are often the lawyers and doctors.

The nobility is made up of Hexagons. At the top of this class is the honorable Polygonal (meaning many-sided).

The priestly order is the highest class and is made up of circular shapes.

Within this hierarchy, women are represented as simple straight lines and, in a satirical view of society in the 1800s, are regarded as inferior, unintelligent, and even dangerous. The size of the female inhabitants is 11 to 12 inches.

While the offspring of the Isosceles Triangles each gain half a degree more than their parents, the offspring of the other shapes – beginning with the Equilateral Triangles and moving up the hierarchy – will gain one side more than their parents. For many-sided shapes, the offspring can gain 50 sides or more. As each masculine shape gains a side, their size increases and they rise up the class totem pole in Flatland.

Gaining extra sides has a disadvantage as well. The more sides a male shape has, the more such shape is at risk of infertility. In order to combat infertility, the inhabitants attempt to skip generations and medically alter the number of sides their children possess in order to combat infertility. The Neo-Therapeutic Gymnasium is the facility which performs such medical experiments. The only problem is that very few offspring survive such experiments.

Turning back to our chosen apostle, A. Square has a dream about a dimension named Lineland and comes to realize that the king of Lineland is close-minded and does not believe that any other place exists. Upon waking, A. Square is asked by his grandson “what is 3D?” A. Square responds that this is not possible, but the question haunts him, particularly following his dream. A Sphere then visits A. Square and proves to him that there is indeed a third dimension. Following this enlightening visit, A. Square proceeds to spend his life trying to prove to everyone that there is a third dimension. Unfortunately, the other inhabitants of Flatland are, much like the king in his dream, narrow-minded and unimpressed with his arguments, and his efforts lead to his imprisonment. A. Square writes this memoir from his prison cell in the hope that someone will, some day, believe his tale.

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I would recommend this book for students of middle school through high school. This book is approachable yet highly instructive and, as such, is very student-friendly. It incorporates many aspects of geometry and would be a great introduction to the different shapes, as well as the concepts of area and volume.

My only reservation in recommending this book to students is that its satirical tone, particularly regarding women, may be lost on, and perhaps confuse or offend a younger reader. In short, I would recommend this book, but would first give the students some background on both the genre of satire and the ideals of society in the 1800s. With that preface, I believe this book would be a success with middle to high school students.

On that note, this book is a great introduction to the genre of satire, possibly something the younger students have yet to encounter. If students are on the lookout for the subtle sarcasm and exaggerations permeating the story, students will gain insight into the prevailing beliefs of society in this time period through the critical lens of the author. This book also introduces literary devices such as personification and symbolism.

Students will benefit from exposure to these literary devices and historical perspectives. The themes of class, gender roles and hierarchies within societies and governments are themes which students will encounter throughout their studies of every time period and therefore provide a great backdrop for history lessons. For example, ideas of authority and rebellion are a great background for a discussion on the American Revolution. The roles and rights of women are another topic that can be introduced with this book.

Flatland is likewise a useful platform for discussions of science and religion. For example, it is a great example of the tension between those in power and those attempting to publish scientific discoveries which the ruling class or government may find inconvenient and a threat to the status quo.

Summing up, I very much enjoyed this book and found it to be very intriguing. I did not expect the plot twist in which A. Square is imprisoned. I find this book to be not only enjoyable and accessible for many ages, but also an invaluable repository of ideas and educational tools relevant to a wide variety of disciplines.
Chapter 1 - Of the Nature of Flatland

Summary - Flatland’s inhabitants are made up of triangles, squares, pentagons, and other shapes on a plane. Solids do not exist in this land. If you place any shape flat on a table and you lower your eye level to it, such shape will appear to be a line. From this perspective, it becomes impossible to see anything but lines.

Definitions:

Two Dimensional - an object that only has width and length, but not height.

Line - a long mark that is straight and has no height.

Plane - a flat surface that extends forever and has no height.

Activities:

- Cut out the following shapes below and place on a table.
- Then lower your head so your eyesight is level with the objects.
- Write down what happens to the shape; what shape does it appear to become?
- Move closer and closer to the shape and describe what appears to be happening to the shape from each perspective.

Additional questions:

Where have you noticed the same type of shape shift occurring on Earth?

Can you apply this activity to the theory (held long ago) that the Earth was flat? Explain.
Chapter 2 - Of the Climate in Flatland

Summary - This Chapter discusses how the inhabitants of Flatland are able to distinguish between north and south. The rain always comes from the north and it is warmer from the south. No one except A. Square knows from where light originates. Houses are built in pentagonal shapes. Very few square houses exist. Triangle buildings are served to hold munitions or are government buildings.

Definitions:

Triangle - a three sided polygon with three straight sides.
Square - a four sided shape with 90 degree angles. All the sides are congruent.
Pentagon - a plane figure with five straight sides and five straight angles.

Questions:
1. A triangle has a sum of 180 degrees. What is the sum of the degrees square?
2. A triangle has a sum of 180 degrees. What is the sum of the degrees in a pentagon?
3. Compare and contrast the vertices of a triangle, square, and pentagon.
4. Describe instances where you have seen each of these shapes in real life.
Worksheet on Vertices and Edges

Calculate the number of vertices and edges for each shape below:

**2 Dimensional Shapes**

![Triangle](image)

Vertices ________  
Edges ________

![Square](image)

Vertices ________  
Edges ________

![Octagon](image)

Vertices ________  
Edges ________

**3 Dimensional Shapes**

![Pyramid](image)

Vertices ________  
Edges ________

![Cube](image)

Vertices ________  
Edges ________

![Tetrahedron](image)

Vertices ________  
Edges ________
**How to graph on a Cartesian Coordinate Plane.**

**Directions:**

If you have a coordinate (x,y), you graph it as follows:

Example: (-3,5)

We begin with the first number in the set. The first number (-3) represents the x axis which means that you will move left or right. Positive numbers mean that you will move right while negative numbers mean that you will move left. (In Flatland terms, right or left is east or west.)

The first number is negative so we will move left three:

![Graph with arrows showing movement left to x-axis and up to y-axis for the coordinate (-3,5).](image)

The second number (5) represents the y axis which means that you will then move up or down. Positive numbers mean that you will move up while negative numbers mean that you will move down. (In Flatland terms, up or down is north or south.)

The second number is positive so we will move up five from the previous spot. Then you will plot this point on the graph by marking a dot.

![Graph showing the coordinate (-3,5) plotted on the Cartesian plane.](image)

This is how you graph numbers.
Graph each of the points below.

Connect the points in order as you graph them.

Remember that an ordinal pair \((x, y)\) represents \((\rightarrow, \downarrow)\), first side to side, then top to bottom.

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<td>(-2, 2 )</td>
<td>16</td>
<td>(1, -17 )</td>
<td>31</td>
<td>(8, 4 )</td>
<td>32</td>
<td>(9, 7 )</td>
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<td>19</td>
<td>(6, -9 )</td>
<td>37</td>
<td>(1, 16 )</td>
<td>38</td>
<td>(-1, 15 )</td>
<td></td>
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<tr>
<td>5</td>
<td>(-4, -12 )</td>
<td>20</td>
<td>(4, -11 )</td>
<td>39</td>
<td>(-7, 14 )</td>
<td>40</td>
<td>(-10, 12 )</td>
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<tr>
<td>6</td>
<td>(-4, -14 )</td>
<td>21</td>
<td>(2, -11 )</td>
<td>41</td>
<td>(-10, 9 )</td>
<td>42</td>
<td>(-7, 6 )</td>
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<td>7</td>
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<td>22</td>
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<td>44</td>
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<td>(3, -6 )</td>
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<td>(1, 4 )</td>
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<tr>
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<td>(3, 5 )</td>
<td></td>
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<tr>
<td>14</td>
<td>(1, -13 )</td>
<td>29</td>
<td>(4, 2 )</td>
<td></td>
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<td>(5, 1 )</td>
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*Snoopy Graph – Math Game Time*

http://www.mathgametime.com/worksheets/snoopy-graph

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Chapter 3 - Concerning the Inhabitants of Flatland

Summary - The lowest class of shapes is the Isosceles Triangles. They have a base of no more than an 1/8 of an inch and the usual size of the other sides is 11 to 12 inches. They are regarded as inferior in intelligence (like the Flatland women); the more dimwitted ones even resemble women, as they look like lines. If an Isosceles Triangle does a good job or has successful war conquests for a long period of time, its base becomes larger and it starts to resemble an Equilateral Triangle. Every generation of offspring gains half a degree more than their ancestors. When an Equilateral Triangle is born he is taken from his parents and given to a childless family of Equilateral Triangles. The whole country is happy when this happens.

The middle class is made up of Equilateral Triangles who are craftsmen. Their offspring will gain one more side so that they will range from Equilateral Triangles up to Octagons. After that, they gain multiple sides.

The professional class is made up of Squares and Pentagons. This class consists of lawyers and doctors.

The nobility is made up of hexagons and greater than hexagons.

The priestly order is the highest class and is made up of circular shapes. There is no true circle in Flatland. The closest a shape may come to a circle is a shape with 1,000 sides.

Definitions-

Vertex - a corner where two edges meet.

Edge - a line segment that joins two vertices.

Types of Triangles

Isosceles Triangle- a triangle with 2 sides that are the same size.

Equilateral Triangle - a triangle with all equal sides and all degrees are equal to 60 degrees.

Other types of Polygons

Hexagon - a 6-sided shape

Heptagon - a 7-sided shape

Octagon- an 8-sided shape

Dodecagon- a 12-sided shape
Directions – Identify the shapes below and state to which class they belong in Flatland. The classes are lower class, middle class, professional, nobility, and priestly order. Please also explain why they belong in these classes. Also state their gender.

1.  
Shape Name_______  
Class  ________  
Gender  ________  

2.  
Shape Name_______  
Class  ________  
Gender  ________  

3.  
Shape Name_______  
Class  ________  
Gender  ________  

4.  
Shape Name_______  
Class  ________  
Gender  ________  

5.  
Shape Name_______  
Class  ________  
Gender  ________  

6.  
Shape Name_______  
Class  ________  
Gender  ________  

7.  
Shape Name_______  
Class  ________  
Gender  ________  

8.  
Shape Name_______  
Class  ________  
Gender  ________  

9.  
Shape Name_______  
Class  ________  
Gender  ________  

Discussion Questions

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Discuss the following topics in class and have the students write an essay on each topic.

1. Begin by discussing the literary device of satire and how the author utilizes this tool to express his opinions of society’s ideals and beliefs in 1800s England. Have the students provide examples of exaggerations and sarcasm that are evident in the book. Discuss what the author is trying to accomplish by writing the book in this genre and how he is using satire a way to effect change in the world by bringing the public’s attention to its flaws, both as individuals and as a whole.

   A. What ideas about each class, and about gender, do you believe the author is attempting to convey to the reader?

   B. How does assigning each class to a type of shape help convey his ideas? For example, what could be the reason behind making Flatland’s women simple straight lines?

   C. What could be the reason behind the author’s decision to represent the lower classes with less complex shapes? What quality does each side of a shape appear to represent? Is it power, wealth, intellect or something else? Explain.

   D. What clue does Flatland’s hierarchy give as to the values of Flatland’s inhabitants?

   E. Why do you believe there is no true circle in Flatland? What idea do you think the author is trying to convey with the absence of a perfect circle?

2. Select an historical figure from any period of time and geographic location. Discuss where in Flatland’s class structure such figure would be ranked. Explain your reasons. Use the following questions as a guideline.

   A. Provide the name, occupation or role, geographic location and the year in which the historical figure you selected lives.

   B. What qualities and abilities do you believe this historical figure has? For example, is this individual very intelligent? Brave? Strategic? Wealthy? Able to create something useful? Talented? Compassionate? Able to heal?

   C. What rank would this historical figure have within Flatland’s hierarchy?

   D. Why would this figure have such a rank?
Summary - Females are straight lines and have the lowest intelligence. The sharper the angle in Flatland, the more dangerous that shape is. A woman is the most dangerous shape in Flatland and can become invisible in a second by turning sideways so that all you see is a point. She is also very brittle. In a second, she may murder all the members of her household and, due to her low intelligence, will not remember it.

Three codes govern women (Abbot, 1992, p.11):

“1. Every house shall have one entrance in the Eastern side, for use of Females only; by which all females shall enter and not by the Men’s door.

2. No Female shall walk in any public place without continually keeping up her Peace-cry, under penalty of death.

3. Any Female, duly certified to be suffering from St. Vitus’s Dance fits, chronic cold accompanied by violent sneezing, or any disease necessitating involuntary motions, shall be instantly destroyed.”

Quiz

Have students answer the following questions. Write them down on the board and have students write them on a sheet of paper.

1. What three rules in Flatland help protect males from females?

2. Do females remember their rage?

3. What is the most dangerous shape in Flatland and why?

4. True or False: there are female pentagons. Explain why you picked true or false.
Summary - In Flatland, there are three ways to tell the difference between shapes. The first is by voice recognition. The second, used more by the lower class, is by feeling the object and feeling how sharp the vertex is. If a shape has multiple sides, then they would count the sides. The third is by sight which only the middle class and above are privileged to possess. The way that a Square can identify the shape of something is to see how fast the vertex and the edge recede into the background. The quicker it recedes, the lower the class of the object, and vice versa. Very little expense is reserved for educating the lower class and that is why they only learn to feel. The upper class spends a third of their life studying and learning to recognize shapes. Very few of the higher class do not graduate school; those that do not graduate and have children will have irregular offspring. These offspring tend to cause rebellion due to their malicious nature.

Given below is an example from the book how the educated are able to discern what type of shape they are looking at.

http://www.geom.uiuc.edu/favicon.ico

http://www.geom.uiuc.edu/favicon.ico
Activity - Break your class into groups of three and have them figure out the number of degrees each shape has.

**Directions:**

Figure out how many degrees each shape has by using the following formula for regular polygons:

\[
\text{Total Sum} = (n-2) \times 180 \quad (n= \text{number of sides})
\]

After figuring out the total degrees, divide the number of degrees by the number of vertices to find out how large each angle is.

1. ![Hexagon](image)
   - Sum of degrees_________
   - Angle Size_____________

2. ![Octagon](image)
   - Sum of degrees_________
   - Angle Size_____________

3. ![ Pentagon](image)
   - Sum of degrees_________
   - Angle Size_____________

4. ![ Square](image)
   - Sum of degrees_________
   - Angle Size_____________

5. ![Octagon](image)
   - Sum of degrees_________
   - Angle Size_____________
Summary - Most of the shapes in Flatland are regular polygons. Shapes are smaller in size at birth and grow as they get older. Irregular polygons are mistrusted and usually the source of malicious activity. At their coming of age, each irregular shape gets inspected and is either killed or must work in a government office; they are not allowed to wed.

Definitions -

Transformations:

Reflection - every point is the same distance from the line it is being reflected from
Rotation - the distance from the center stays the same and the object is turned around a center.
Translation - moving each of the vertices the same distance away by x and y units.
Dilation - each side becomes larger or smaller while retaining its original geometric shape.

Proportions:

Congruent - two objects have the same shape or same size.
Similar – two objects have the same shape but are different sizes while still geometrically proportional.
Similar Triangles

Similarity in Triangles

Similar Triangles Worksheet (1 of 2)

Similar Figures Worksheet

Transformations

Transformations Worksheet (2 of 3)

Transformations Worksheet (1 of 3)

Transformations Worksheet (3 Pages)

Dilations

Congruence & Similarity Worksheet: Dilations

Congruence

Congruent Triangles: With Transformations

Congruence Worksheet (1 of 3)

Chapter 8 - Of the Ancient Practice of Painting
Summary - A Pentagon known as Chromastistes first painted himself, then painted his slaves and family members. This trend caught like wildfire and within two months almost everyone in Flatland was painted in different colours. Different classes had were represented by certain colours. The only two shapes not to adapt were women, because they had no width to be painted, and the priestly order, since they were, presumably, above such vanity.

Chapter 9 - Of the Universal Colour Bill

Summary - An Irregular Circle proposed a bill that all shapes were to be painted. The Circles and Lines were to be painted red on their mouths and green on the rest of them. The bill was allegedly proposed in order to favor women since, if the women were painted in the colors of the circles, a much higher rank, they would be treated as such. The Irregular Circle proposing this bill was manipulative, however, and had many ill intentions. For example, this bill would have all shapes painted which would obviate the practice of sight recognition, a practice utilized by the circular households, as well as mathematics in general. Sight recognition and the study of mathematics suffered during this period due to the Flatland inhabitants abandoning the identification of shapes by sight and mathematical principles, and instead classifying shapes by color alone.

Trigonometric Ratios

Sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{a}{c}

Cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}

Tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{c}
Activity - Have students break into groups of two and three and work together to find the missing angles or missing sides of each shape below.

1. \(\triangle ABC\) with sides 15, \(a\), and \(b\), and \(\angle C = 34\) degrees.

2. \(\triangle ABC\) with sides 13, \(a\), and \(b\), and \(\angle C = 32\) degrees.

3. \(\triangle ABC\) with sides 10, \(a\), and \(b\), and \(\angle C = \) unknown.

4. \(\triangle ABC\) with sides 13, \(a\), and \(b\), and \(\angle C = \) unknown.

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Chapter 10 - Of the Suppression of the Chromatic Sedition

Summary - When the turmoil in Flatland was at its highest peak, Isosceles Triangles wiped out an army of polygons. Wives were pressuring their circular husbands to agree to the Colour Bill since it would improve their rank. When they refused, their wives went mad and killed their husbands, as well as their children. Over 23 Circles died during this period. It looked like all was lost and the Circles would have to concede to the Colour Bill.

As luck would have it, an Isosceles Triangle of 4 degrees, having been in pursuit of a noble female polygon for some time, robbed a tradesmen store and painted himself in the colors of a Dodecagon. Since shapes identified each other by color, the unwitting female polygon was tricked into marrying the disguised Isosceles Triangle and consummating the wedding. When she later discovered her husband’s true identity, she killed herself. All the women in Flatland became outraged and began to see the Colour Bill in a different light.

In the meantime, Pantocyclus, the Chief Circle, was pressured by over 120,000 Isosceles Triangles – who would no doubt benefit from the Colour Bill as it could only improve their rank – into conceding thereto. Pantocyclus recognized both the opportunity his speech would offer and requested for the leader of the Sedition Chromatistes to be present. Once everyone was assembled, Pantocyclus acquiesced to the Colour Bill and went on to provoke the crowd by stating that, as a result of the bill, the class system would cease to exist, the Triangles would become regular and upper class children would no longer move up in the ranks. Some members began to murmur with alarm. Pantocyclus continued by addressing the women, noting that no marriage would be annulled due to trickery based on color misrepresentation. The women, angered by this prospect of marriage by fraud, began to attack and kill the members of the Sedition, including their leader whom Pantocyclus had conveniently invited. Now leaderless, the sedition turned on each other. So ended the rebellion based on the Colour Bill. For many years thereafter, such was the mistrust that army member’s houses were consistently checked for signs a new rebellion. Colour is now only mentioned to the privileged few who obtain higher education.

Definitions:

Perimeter - the distance around an object calculated by adding all the sides

Apothem of the polygon - the height of a triangle between the center and two consecutive vertices of the polygon.

Central angle of a regular polygon - an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon.

Area of a Regular Polygon- is the product of the apothem a and the perimeter P

\[ A = \frac{1}{2} \cdot a \cdot P \text{, or } A = \frac{1}{2} \cdot a \cdot (ns) \]
Name: 
Date: 

Directions - Calculate the area of each polygon below. Remember to use trigonometry functions to find the apothem or the side length.

1. Apothem = 4 ft.
   
   Perimeter - ________
   Apothem - ________
   Area - ____________

2. Apothem = 7 ft.
   
   Perimeter - ________
   Apothem - ________
   Area - ____________

3. Side length = 6 ft
   
   Perimeter - ________
   Apothem - ________
   Area - ____________

4. Side length = 4 ft
   
   Perimeter - ________
   Apothem - ________
   Area - ____________

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Chapter 11 - Concerning Our Priests

Summary - Circles are the priests. They are the highest educated class. They decide the functions of the lower ranking shapes and how things are run. Circles have a tendency to become infertile. They have a difficult time producing sons as they gain more sides. If they reproduce successfully, their male offspring can gain 50 or 100 sides. Polygons figured out a way to combat this problem by skipping generations. This was carried out by sending their male sons (before the age of a month) to The Circular Neo-Therapeutic Gymnasium where their sides are broken and reset. If successful, many sides can be added and the polygon can resemble a circle. This method of course has its dangers: only 1 out of every 10 children survives the process.

Discussion Questions:

Have the students break into groups of three or four, discuss and write an essay on the following topic:

A major theme in Flatland is the class system and how difficult it is for the lower class to move up within the ranks. Discuss how this theme reflects the values of society of England in the 1800s.

(a) What attributes and contributions did society in 1800s England appear to value the most?

(b) What did society in 1800s England appear to value the least?

(c) How did Flatland’s ruling class ensure that the class system was maintained?

(d) What effects did the deprivation of education for the lower class have on the class system and society as a whole?

(e) Discuss how the extreme measures taken by the inhabitants of Flatland to raise their class and that of their offspring (e.g., risking their children’s lives to undergo experiments at the Gymnasium or happily giving up their children to childless members of a higher class) was mirrored in 1800s England. What strategy and/or risks did people take to achieve a higher status in society? Did climbing the socio-economic ladder appeal to certain classes more than others?

(f) Are such measures to move up the ranks (described in your response to question (e) above) evident at all in modern society in the United States? Explain.

(g) Are measures taken by the higher classes in Flatland to control the lower classes, such as controlling access to education and the voice of the people, evident at all in the modern U.S.?

(h) What attributes and contributions does modern society in the U.S. appear to value the most? What does it appear to value the least? Explain.

(i) List the measures taken by the founders of the U.S. to protect American citizens from the oppression at the hands of a powerful ruling class. Were these measures successful? Explain.
Chapter 12 - Of the Doctrine of Our Priests

Summary - Pantocyclus required any irregular shapes to be fixed by going to their respective hospitals. The alternative was spending the rest of their days in jail. As a result, shapes started blaming bad decisions or misdeeds on the temperature changing their sides slightly and causing them to become irregular.

Irregular women were not allowed to wed. The higher the polygon, the less they looked into the ancestry of women and still wed. Women were deemed unintelligent and not allowed to learn how to read, write, or do mathematics. Women were deemed emotional shapes and incapable of reason.

Discussion Questions:

Have the students break into groups of three or four, discuss and write an essay on the following topic:

1. Women in Flatland are viewed as inferior and were prevented from receiving an education. Discuss how this perspective of women reflected the predominant societal view of women in England in the 1800s and, if at all, in the United States from the 19th century through the present time.

   (a) What were the primary roles of women in 1800s England?
   (b) Who decided what their roles would be?
   (c) Did these roles impact how women were regarded by society?
   (d) Did a woman’s economic class alter her role or her ability to decide what her role would be?
   (e) Explain how the role of women has changed over time from England in the 1800s through the 19th-20th centuries in the U.S. and today.
   (f) How has women’s access to education played a role in these changes?
   (g) Do you believe Flatland’s views of women - as emotional and incapable of reason – are present anywhere today in modern U.S. society?
   (h) Why were the women in Flatland considered the most dangerous, although they were the most inferior? Explain.
   (i) What governmental, political and societal protections have evolved over time to ensure that women are treated as equals? Have these measures worked? Explain.
   (j) What do you believe the author is trying to accomplish by representing women in such a manner? Recall our discussion of satire. How is the author using the genre of satire to reflect society’s ideals and, possibly, affect awareness on the part of his readers? Could this awareness lead to change?

2. What does Flatland’s treatment of irregular shapes say about the need for conformity in Flatland? Why was conformity so important? What would happen if irregular shapes were not monitored, fixed and controlled? How would the hierarchy be impacted?

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Chapter 13 - How I had a Vision of Lineland
Summary - On the last day of the 1999th era, a Square has a dream of lines moving left and right. He tried getting an answer as to this occurrence, but to no avail. Only when he put his mouth into Lineland did he get a response from the King.

The inhabitants of Lineland only move straight, cannot pass one another and are neighbors for life. They can only see a point in front of them. The males have two voices - a Bass and Tenor - at each of their ends. Marriages can occur over any distance by the lines singing in harmony with an individual female. Males wed two females so they have perfect harmony consisting of Bass, Tenor, Soprano, and Contralto. The offspring that can be expected are either one male or two females at a time. If there were not as many females born Lineland would cease to exist.

Chapter 14 - How I Vainly Tried to Explain the Nature of Flatland

Summary - The king of Lineland is 6.457 inches tall. The people of Lineland are not allowed to move in close proximity with another; the punishment for doing so is death. Since they can only see a point, and cannot see past one another, they base distance by voice and sound. A Square tries to prove to the ignorant king of Lineland that there is a land called Flatland. The king of Lineland responds angrily and attempts to kill a Square. A Square then wakes up from his dream.

Definition -

Distance - how far two objects are apart.
Activity: Have students get up and walk to different places in the room. Have them figure out the distance of one another by doing Pythagorean Theorem.

Example

The blue point is 6 units left of the black point. The black point is 5 units right of the blue point.

\[ A^2 + B^2 = C^2 \]
\[ 6^2 + 5^2 = C^2 \]
\[ 36 + 25 = C^2 \]
\[ 61 = C^2 \]
\[ \sqrt{61} = C \]

Once completed have students complete the following worksheet found on the following link.


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Chapter 15 - Concerning a Stranger from Spaceland
Summary – A. Square is teaching his adopted grandson, Geometry, and shows him what $3^2$ means. His grandson then replies well then $3^3$ must mean something. A. Square gets angry and tells his grandson to go to bed. Then A. Square feels a presence in the room. It is a stranger. The stranger introduces himself to A. Square and his wife. He says that he is a circle made of many smaller circles and his news is such that he can only tell A. Square in private.

Chapter 16 -How the Stranger Vainly Endeavored to Reveal in Me in Words the Mysteries of Spaceland

Summary - The perfect circle tries to prove to A. Square that there is a third dimension consisting of height, breadth, and thickness. A. Square does not believe the stranger. When the stranger then vanishes out of Flatland - the same way that A. Square vanished out of Lineland - he still does not believe it. He tries to reason to him with arithmetic progression 1, 2, 4, 8.

Definitions:

Perfect Squares - the product of a rational number multiplied by itself.

Perfect Cube - the product of three integers multiplied by themselves.

Have students watch the following videos from Khan Academy:

Simplifying square roots
Simplifying a cube root

After watching those videos, have students complete the following worksheets for homework:

Simplifying square roots
Simplifying cube roots and other radicals

Chapter 17 -How the Sphere, Having in Vain Tried Words, Resorted to Deeds
Summary – A. Square tried to kill the Sphere but the Sphere left the dimension. The Sphere tried to convince the Square that he is from a different dimension. The sphere went into the cupboard and took out a tablet and reappeared into Flatland and brought the tablet back. A. Square still did not believe. The Sphere then went out of the dimension and touched the stomach of A. Square. When the Sphere reappeared, A. Square tried to pin the Sphere against the wall.

Chapter 18 - How I came to Spaceland, and What I saw there

Summary - The Sphere takes A. Square out of Flatland into a three-dimensional world. There A. Square sees the many faces of what makes up a solid. He is amazed and, at first, was going to worship the Sphere, but the Sphere made sure he did not. The Sphere took A. Square to view what was happening at the state building; a Square saw his brother getting imprisoned for being “Apostle of the Gospel Three Dimensions.” The Sphere manifested himself in Flatland and the policemen tried to catch him but could not. Evidently this has happened several times on each day of the new millennium. The high council ordered the police to be exterminated for their failure, and to ensure the conspiracy was kept from the rest of the inhabitants of Flatland, and ordered a Square to be imprisoned for life in order to keep the secret.

Chapter 19 - How, Though the Sphere Shewed Me Other Mysterious of Spaceland, I still Desired More; and What Came of It

Summary - The Sphere shows A. Square the various solids that exist. A. Square becomes ecstatic and requests to see a fourth dimension, then a fifth dimension. The Sphere tells him that no such dimension exists. After a while, the Sphere brings A. Square back home to Flatland.

Chapter 20 - How the Sphere Encouraged Me in a Vision

Summary – A. Square has a dream where he and the Sphere visit Lineland to convince the King of a third dimension. The king is ignorant and only believes that he and his world are the only thing that exists.

Chapter 21 - How I Tried to Teach the Theory of Three Dimensions to My Grandson, and with What Success.

Summary – A. Square tried to convince his grandson about a third dimension. He was trying to explain the concept of how upward - as opposed to northward - would make the third dimension. The grandson, aware of the Resolution of the Council being conducted outside, pretended to have been joking. He said he never believed in anything else but Flatland.

Chapter 22 - How I Then Tried to Diffuse the Theory of Three Dimensions by Other Means and the of the Result.

Summary - 11 months have passed. A. Square eventually describes what he saw with the Sphere. He is immediately sent to the Council but is unable to prove anything. He is promptly jailed.

7 years have passed. Every so often A. Square gets to spend time with his jailed brother. He writes this memoir hoping eventually to convince someone of his account.
Solid - a three dimensional object.
Three Dimensions - width, depth, and height

**Activity:**

Have students watch the following video on types of Solids:

https://www.youtube.com/watch?v=hb2nMfHU_DY

Have students work on the following worksheets:

1. Identifying the shape as either second or third dimension

2. Counting the Faces, Edges, and Vertices

3. Matching second-dimensional shapes with their third-dimensional shapes counterparts

4. Counting the faces, edges and vertices of third-dimensional shapes and naming them

**Discussion Questions**
Discuss the following topics and have students write an essay on the same:

1. In Flatland, the circular figures made the decisions and the other inhabitants were required to listen to them without question. Select an historical period where such system of power and authority was evident.
   1) Describe the structure of the government, classes and society in your chosen place and time.
   2) What categories of people had the strongest voice? The weakest? Why?
   3) What was the role of women in this era?
   4) What role did education play in this era? Who had access to an education?
   5) Were there any exhibitions of resentment on the part of certain people towards this system, or were there any efforts to change the same? What was the outcome?
   6) How long did this form of government exist? Is it still around today? If not, what do you believe caused its downfall and what government system replaced it?

2. The inhabitants of Flatland were expected to obey without question their elders as well as those in higher positions of power.
   1) Discuss whether this perception of one’s elders in general – as authoritative and wise – is present in modern day society. Feel free to make comparisons between the U.S. and any other cultures.
   2) Have the views of the elderly changed over time? Do you believe they are more or less highly regarded as time goes on?
   3) Did the conversation between a Square (a grandfather) and his grandson reflect this view (in Flatland) of elders as individuals with authority?
   4) Did the subsequent treatment of a Square at the hands of the Council reflect this view?
   5) How did the jailing of A. Square by the Council reflect the importance of power and class over respect for the elderly, wisdom and truth?
   6) Describe an instance in history where truth and enlightenment proved too dangerous for those in power at the time, and was, therefore, suppressed.
      a) How did the particular truth, enlightenment or facts in the scenario you selected threaten the status quo? What consequences would the publication and acceptance of such truths have on that society, that ruling class and the course of history?
      b) What was the age, educational background or religion, if applicable, of the person or group attempting to shed light on such truth or share their beliefs? What was their economic or societal class status? Did their background determine their treatment?
      c) What was the ultimate outcome?

3. A. Square is initially hesitant, doubting and even angry to point of violence in the face of the Sphere’s revelations as to a third dimension.
A. Why do you believe A. Square acted in this manner initially upon the Sphere’s visit?

B. Why did A. Square become angry at his grandson’s question?

C. What ideals and fears in 1800s English society is the author trying to mirror with A. Square’s reaction?

D. Do you recall a time in history where society had a similar reaction to a scientific fact?

4. A. Square, once he believes the Sphere, is ecstatic. He is so excited about the third dimension he cannot wait to see more. He wants to see a fourth, even a fifth dimension, but the Sphere tells him that none exists.

A. The third dimension was obviously too much for Flatland’s inhabitants to handle. Their reaction to a fourth dimension would likely not bode well for A. Square, already imprisoned.

i. What reaction would 1800s English society have to the concept of additional dimensions? Obviously they were familiar with the third dimension. What about the fourth (time), or even a fifth?

ii. Would they be excited or angry?

iii. Does the author appear to believe that society was sufficiently open-minded to accept new scientific discoveries?

B. Select a scientific discovery which was new in the late 19th century.

i. What was society’s initial reaction to such discovery?

ii. How did this discovery change society?

iii. Did it lead to additional discoveries?

iv. How was the inventor of such discovery treated by society?
